

# The 79th KPPY Combinatorics Seminar

Organized by S.Bang, M.Hirasaka, T.Jensen, and M.Siggers

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Natural Sciences Building, Room 313

Department of Mathematics, Kyungpook National University

## Program

11:00 - 11:50 **Suyoung Choi** Ajou University  
Wedge operations in the classification problems of (real) toric varieties

12:00 - 1:30 **Lunch**

1:40 - 2:30 **Tommy R. Jensen** KNU  
Construction of graphs without 3-flow

2:40 - 3:30 **Qianqian Yang** USTC  
An application of Hoffman graphs for spectral characterizations of graphs

4:00 - 4:50 **Jack Koolen** USTC  
On graphs with a few eigenvalues

5:00 - 5:50 **Hyonju Yu** POSTECH  
Harmonic Distributions for Equitable partitions of a Hypergraph

6:30 - 8:30 **Banquet**

## Abstracts

### Suyoung Choi

Wedge operations in the classification problems of (real) toric varieties

A fundamental result of toric geometry is that there is a bijection between (real) toric varieties and fans. More generally, it is known that some classes of manifolds having well-behaved torus actions, say toric objects, can be classified in terms of combinatorial data containing simplicial complexes.

In this talk, we investigate the relationship between toric objects over a simplicial complex  $K$  and those over the complex obtained by simplicial wedge operations from  $K$ . Our result provides a systematic way to classify toric objects associated with the class of simplicial complexes obtained from a given  $K$  by wedge operations.

As applications, we discuss how to classify smooth toric objects of small Picard numbers and how to consider their projectivity.

### Tommy R. Jensen

Construction of graphs without 3-flow

A *nowhere-zero  $k$ -flow* in  $G = (V, E)$  is a pair  $(\vec{G}, \psi)$ , where  $\vec{G}$  is an orientation of  $G$  and  $\psi : E \rightarrow \{-k + 1, \dots, -1, 1, \dots, k - 1\}$  satisfies *Kirchhoff's Law* of flow conservation:

$$\sum_{\{e \in E : h(e)=v\}} \psi(e) - \sum_{\{e \in E : t(e)=v\}} \psi(e) = 0 \quad \text{for all } v \in V.$$

We characterize constructively the class of graphs that do not allow a nowhere-zero 3-flow.

### Qianqian Yang

An application of Hoffman graphs for spectral characterizations of graphs

In this talk, we will show that the 2-clique extension of the  $(t \times t)$ -grid is determined by its spectrum if  $t$  is large enough. In order to show this, we use Hoffman graphs as our main tools. Hoffman graphs were introduced by Woo and Neumaier (1995) and were derived from Hoffman's idea in his 1977 paper. Later, Kim, Koolen and

Yang (2016) and Koolen, Yang, Y. (201?) developed further the theory of Hoffman graphs and proved that if  $t$  is very large, the 2-clique extension of the  $(t \times t)$ -grid is a 2-fat  $\{\text{K}_4, \text{K}_5, \text{K}_6\}$ -line Hoffman graph. Based on this fact, we show that the 2-clique extension of the  $(t \times t)$ -grid is determined by its spectrum if  $t$  is large enough. This is joint work with Aida Abiad (Maastricht) and Jack Koolen (USTC).

### **Jack Koolen**

On graphs with a few eigenvalues

Graphs with at most three distinct eigenvalues are well-studied. A main eigenvalue of a graph is an eigenvalue with an eigenvector not orthogonal to the all one vector. A plain eigenvalue is an eigenvalue with an eigenvector orthogonal to the all-one vector. Note that usually a bi-regular graph with three distinct eigenvalues has two main and two plain eigenvalues. In this talk I will consider the class of graphs with two main and two plain eigenvalues. A graph in the switching class of a non-trivial regular two-graph has this property. It was shown by Van Dam, K. and Xia that such a graph can have as many distinct valencies as you prefer. This is joint work with Sakander Hayat and Muhammad Javaid.

### **Hyonju Yu**

Harmonic Distributions for Equitable partitions of a Hypergraph

We provide general criteria for orthogonal arrays and  $t$ -designs on equitable partitions of a hypergraph by exploring harmonic distributions. Generalized harmonic weight enumerators for complex-valued functions of hypergraph are introduced and applied to eigenfunctions of the adjacency matrix of hypergraph. Using this, expressions for harmonic distributions are established for every cell of an equitable partition of hapergraph. Moreover, for any given cell in the partition, the strength of the cell as an orthogonal array is explicitly expressed, and also a characterization of a  $t$ -design of that cell is established.

This is joint work with Hyun Kwang Kim, Jongyoon Hyun.