Industrious Number Theory 4

February 16 – 18, 2012
at
Seminar Room 313-2
College of Natural Sciences
Kyungpook National University

Organized by
MOON Hyunsuk (Kyungpook National University)
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Time Table

	Thu 02/16	Fri 02/17	Sat 02/18
1	Choi SY	Akatsuka	Hattori
2	Choi DH	Yhee	Chang
	Lunch	Lunch	Lunch
3	Aoki	excursion	Ozeki
4	Ohara	excursion	Lee DU
5	Jun	excursion	Park
6	Lee JY		

Program

Feb. 16 (Thu)

9:30–10:30   CHOI SoYoung (Dongguk Univ.)
Properties of a certain basis for the space of weakly holomorphic modular forms in higher level cases

10:40–11:40   CHOI Dohoon (Korea Aerospace Univ.)
Central values of modular L-functions mod p

11:40–13:30   Lunch

13:30–14:30   AOKI Miho (Shimane Univ.)
On the Coates-Sinnott conjecture for totally real or CM fields

14:40–15:40   OHARA Mariko (Tohoku Univ.)
Rational elliptic surfaces related to Beilinson's Tate conjecture

15:50–16:50   JUN Byungheup (KIAS)
Reciprocity of Todd Series, Dedekind sum and special values of zeta functions

17:00–18:00   LEE Jungyun (KIAS)
Special values of Hecke L-function of real quadratic fields and class number problem of real quadratic fields

Feb. 17 (Fri)

9:30–10:30   AKATSUKA Hirotaka (Kyushu Univ.)
Double zeta functions constructed by absolute tensor products

10:50–11:50   YHEE Donggeon (Seoul National Univ.)
A conjecture of Gross and Zagier and nontrivial examples of Shafarevich-Tate group

Feb. 18 (Sat)

9:30–10:30   HATTORI Shin (Kyushu Univ.)
Canonical subgroups via Breuil-Kisin modules

10:50–11:50   CHANG Seunghwan*, LEE Yoonjin (Ewha Womans Univ.)
Galois characters arising from Drinfeld modules

11:50–14:00   Lunch

14:00–15:00   OZEKI Yoshiyasu (Kyushu Univ.)
Torsion representations arising from $(\varphi, \hat{G})$-modules

15:20–16:20   LEE Dong Uk (POSTECH)
Some new cases of the Mumford-Tate conjecture

16:40–17:40   PARK Jeehoon (POSTECH)
p-adic Weil representations of $\mathrm{GL}_2(\mathbb{Z}_p)$

– *: Speaker

Abstracts

AKATSUKA Hirotaka
Title: Double zeta functions constructed by absolute tensor products
Abstract: In 1990s Kurokawa introduced a notion of the absolute tensor product. This constructs a meromorphic function (a multiple zeta function) from some ordinary zeta functions. By definition, zeros and poles of our multiple zeta function are given by sums of zeros or poles of the ordinary zeta functions. In this talk we will discuss Euler product expressions for our double zeta functions constructed by the Riemann zeta-function and its Euler factors.

AOKI Miho
Title: On the Coates-Sinnott conjecture for totally real or CM fields
Abstract: Let k be a totally real number field and F be a totally real or CM field that is a finite abelian extension over k. The Coates-Sinnott conjecture asserts that a certain element constructed by the special values of the partial zeta functions of k annihilates the K-groups $K_{2n}(\mathcal{O}_F)$, where $\mathcal{O}_F$ is the ring of the algebraic integers of F. In this talk, we assume that the Quillen-Lichtenbaum conjecture holds, and give a simple proof of the p-part of the conjecture for a totally real or CM field F and odd prime numbers p satisfying $p \nmid [F : F \cap k_\infty]$, where k_∞ is the cyclotomic $\mathbb{Z}_p$-extension of k.

CHANG Seunghwan, LEE Yoonjin
Title: Galois characters arising from Drinfeld modules
Abstract: We characterize Galois characters that arise from Drinfeld modules over finite fields in the sense that they can be constructed by the Galois action on the torsion points of Drinfeld modules.

CHOI Dohoon
Title: Central values of modular L-functions mod p
Abstract: Let p be a prime more than 5. In this talk. we will talk a bout non vanishing modulo p for central values of a family of modular L-functions twisted by quadratic characters.

CHOI SoYoung
Title: Properties of a certain basis for the space of weakly holomorphic modular forms in higher level cases
Abstract: We construct a certain basis for the space of weakly holomorphic modular forms for $\Gamma_0^+(p)$ when the genus of $\Gamma_0^+(p)$ is zero, where p is a prime number. Moreover we investigate properties of the

basis elements. Precisely we show that the Fourier coefficients of the basis elements have a duality and that they are integral and that the basis elements satisfies a generating function. Moreover we prove the Fourier coefficients of these basis elements are often highly divisible by prime p in the case that p is small.

HATTORI Shin
Title: Canonical subgroups via Breuil-Kisin modules
Abstract: The overconvergence of the canonical subgroup of the universal abelian variety is one of the key ingredients of the theory of overconvergent p-adic Siegel modular forms. In this talk, I will explain how to show the overconvergence of the canonical subgroups of any level with expected properties (namely, on the Frobenius Kernel, the Hodge-Tate kernel, the degree of the quotient, their total isotropicity, family and anti-canonical isogenies) via the Breuil-Kisin classification and the ramification theory of finite flat group schemes, including the case of $p = 2$.

JUN Byungheup
Title: Reciprocity of Todd Series, Dedekind sum and special values of zeta functions
Abstract: 1-d Todd series is the generating function of the Bernoulli numbers. Classically, the infinite order differential operator given as Todd series evaluated by the unit vector field yields one side of the celebrated Euler-Mclaurin formula. Later in later half of 20th century, Todd series fit to characteristic classes, namely the Todd class complements the Chern character to make the Grothendieck-Hirzebruch-Riemann-Roch theorem. Standard techniques of toric geometry such as dual cone construction, fans, etc further generalize and relate the earlier two directions to sums of values of lattice function in higher dimension. This is crucial step in counting the number of geometric quantizations via the moment map in terms of toric geometry or geometry of lattice polytopes.

In this talk, we aim to some application of these ideas to Dedekind sum and special values of zeta functions arising from cones. In particular, all the Dedekind sums in 2d are recovered as coefficients of Todd series attached to a cone, so that they satisfy the reciprocity coming from the fan structure of the underlying cone. These values are, via Zagier's asymptotic formula, related to the zeta values at the negative integers. This is a joint-work with Jungyun Lee.

LEE Dong Uk

Title: Some new cases of the Mumford-Tate conjecture

Abstract: We prove the Mumford-Tate conjecture for certain abelian varieties which have semistable reduction of toric rank one at a finite place and whose endomorphism ring contains a totally real field. This generalizes a recent result of Hall.

LEE Jungyun

Title: Special values of Hecke L-function of real quadratic fields and class number problem of real quadratic fields

Abstract: In real quadratic field case, we don't know whether there are infinitely many real quadratic fields or not. This is mainly due to the uncontrollable regulators. Thus the development of the class number problem of real quadratic fields has been made by restricting to some families where the regulator part is controlled. The best known family is Richaud-Degert type. In this family, as in imaginary quadratic fields case, it is well known that there are only finitely many real quadratic fields with given class number. Moreover under the assumption of the generalized Riemann hypothesis, we can obtain explicit upper bound of discriminant of Richaud-Degert type with given class number. Recently, Biro, Dongho Byeon and the author developed a way to obtain explicit upper bound of discriminant for some families of Richaud-Degert type with class number one, not assuming the generalized Riemann hypothesis. For this we use a polynomial behavior of the values of Hecke-L-function at $s = 0$.

In this talk, we generalize Richaud-Degert type and enlarge the families of real quadratic fields for which we deal with class number problem. We consider the following family of real quadratic fields $K_n = \mathbb{Q}(\sqrt{f(n)})$ for $f(x) \in \mathbb{Z}[x]$ and an ideal $\mathbf{b}_n$ of K_n such that

$$\mathbf{b}_n^{-1} \sim [1, \omega(n)]$$

and

$$\omega(n) - 1 = [a_0(n), a_1(n), \cdots, a_{s-1}(n)]$$

with $a_i(x) \in \mathbb{Z}[X]$. We noticed that the family K_n satisfying the above condition has the regulator in control because of fixed length of continued fraction. Thus we can conclude there are only finitely many K_n with given class number in the family. Moreover we find that if $a_i(x)$ are all polynomial function with a degree $\leq d$ then special values $L_{K_n}(0, \chi_n, \mathbf{b}_n)$ are quasi polynomial of degree d. (This is joint work with Byungheup Jun.)

OHARA Mariko

Title: Rational elliptic surfaces related to Beilinson's Tate conjecture

Abstract: In this talk, we present examples of a rational elliptic surface over a field such that the complement of all fibers of split type I satisfies Beilinson's Tate conjecture for the second K-group but the boundary map arising from the localization sequense is not surjective. We consider the case that the base field is positive characteristic and transcendental over its prime subfield.

OZEKI Yoshiyasu

Title: Torsion representations arising from $(\varphi, \hat{G})$-modules

Abstract: The notion of a $(\varphi, \hat{G})$-module is defined by Tong Liu in 2010 to classify lattices in semistable representations. In this talk, we study a "maximal" ("minimal") object of a $(\varphi, \hat{G})$-module by using the theory of étale $(\varphi, \hat{G})$-modules, essentially proved by Xavier Caruso, which is an analogue of Fontaine's theory of étale (φ, Γ)-modules. Non-isomorphic two maximal (minimal) objects give non-isomorphic two torsion p-adic representations.

PARK Jeehoon

Title: p-adic Weil representations of $\mathrm{GL}_2(\mathbb{Z}_p)$

Abstract: We introduce the notion of compatible monomial family of $\mathrm{GL}_2(\mathbb{Z}_p)$ and construct p-adic admissible Weil representation of $\mathrm{GL}_2(\mathbb{Z}_p)$. We will explicitly describe the action of the Weyl element on our p-adic Banach space.

YHEE Donggeon

Title: A conjecture of Gross and Zagier and nontrivial examples of Shafarevich-Tate group

Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$ of conductor N and K be an imaginary quadratic field satisfying Heegner condition. We may define a Heegner point P_K of $E(K)$. If P_K has infinite order, then the L-function $L(E, s) = \sum_{n=r}^{\infty} a_n(s-1)^n$ has simple zero at $s = 1$. In this case, there are two formula about the leading coefficient a_1 ; BSD II and Gross-Zagier. Combining both formula, we have new conjecture ([G-Z])

$$[E(K) : \mathbb{Z}P_K] = c \cdot m \cdot |\mathrm{III}(E/K)|^{\frac{1}{2}}, (1)$$

where c is the Manin constant of E and m is the product of Tamagawa numbers of E at prime divisors of N. The right-hand side should be nontrivial if $E(\mathbb{Q})$ has a nontrivial torsion point. In this talk, I will

sketch (1) and introduce some nontrivial example of Shafarevich-Tate group.

Main Reference.

[G-Z] B. H. Gross and D. B. Zagier, *Heegner points and derivatives of L-seires,* Invent. math., **84** (1986), 225–320.