Design Study on Slow Beam Extraction in the 1 GeV Rapid Cycling Synchrotron at the PEFP

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We studied third-order resonance effects in a slow extraction system for a future 1-GeV proton synchrotron the Proton Engineering Frontier Project. This paper describes the properties of the optics design and optimizations for slow extraction in the synchrotron. The effects of the third-integer resonance in Rapid Cycle Synchrotron were investigated. We performed beam tracking simulations to observe the motions of particles in phase space by using the Methodical Accelerator Design 8 program. Finally, we performed beam trace simulations at the designed slow extraction line and showed that the proton beam could be stably extracted by using the RCS.

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I. INTRODUCTION

The Proton Engineering Frontier Project (PEFP) is a 100-MeV proton linear accelerator development project. The rapid cycling synchrotron (RCS) is a future accelerator for the PEFP [1]. The basic parameters for the RCS are chosen to achieve a high current synchrotron. The 100-MeV proton beam is ramped up to 1 GeV for extraction at the RCS. The number of proton per bunch is \(2 \times 10^{13}\), and the RCS has a fourfold symmetry lattice. Several upgrade options are expected to increase the final proton energy and the beam current.

The slow beam extraction system due to sextupoles in the RCS was investigated. Generally, a slow beam extraction system uses the third-integer resonance. The main considerations to generate the third-integer resonance are the strengths and the locations of the sextupoles. The size of the stable triangle region in the phase space depends on the strength of the sextupoles. For stable beam extraction, we designed the slow beam extraction line at the RCS in the PEFP. The slow beam extraction line consists of four bump magnets, an electrostatic septum and five septum magnets. The particle tracking simulation in phase space by using the MAD8 program was investigated to slowly and continuously extract the proton beam in the RCS. The details of the slow extraction mechanism are presented in the following.

II. EFFECTS OF THE THIRD-INTEGER RESONANCE IN THE RCS

A betatron tune shift due to the sextupoles was used to generate the third-integer resonance at the RCS in the PEFP. In this section, we will discuss the third-integer resonance effects due to the sextupoles at the RCS. Generally, the second- or the third-integer resonances are used for slow extraction systems. The third-integer resonance mechanism was selected for the RCS in the PEFP. For the third-integer resonance, the equations of motion can be estimated by using the following Hamiltonian [2]:

\[
H = (\nu x - \frac{m}{3})J + RU_{3m} \cos 3\phi \implies H = \delta J + \epsilon J^{3/2} \cos 3\phi, \tag{1}
\]

where \(\delta = \nu x - \frac{m}{3}\) and \(\epsilon = RU_{3m}/J^{3/2}\) are constants. The third-integer resonance separatrix and tori of the time independent Hamiltonian (Fig. 1) were plotted, and the actions \(J_{SFP}\) and \(J_{UFP}\) were marked. Also, the resonance line is \(m/3 = 13/3\), and the reference frequency is 1.170 MHz. The contours (Fig. 2) were plotted by using the Hamiltonian in \(J - \phi\) space.

Fixed points are given by

\[
\frac{dJ}{ds} = -\frac{\partial H}{\partial \phi} = 3\epsilon J^{3/2} \sin 3\phi, \tag{2}
\]

\[
\frac{d\phi}{ds} = \frac{\partial H}{\partial J} = \delta + \frac{3}{2}\epsilon J^{3/2} \cos 3\phi. \tag{3}
\]

When Eqs. (2) and (3) are zero, a particle will stagnate in phase space, lying on a “fixed point”. Therefore Eq.
(2) satisfies the condition
\[ \sin 3\phi = 0 \] (4)
so that \( dJ/ds = 0 \). Then, we can write \( \cos 3\phi = \pm 1 \) in Eq. (3). However, Eq. (3) should be zero so that the condition is
\[ \cos 3\phi = -1 \] (5)
These conditions on \( \phi \) defines three fixed points at
\[ \phi = \pi/3, \ 3\pi/3, \text{ and } 5\pi/3. \] (6)
The amplitude \( J \) is given by
\[ J_{FP} = (2\delta/3\epsilon)^2. \] (7)

Figure 3 shows the amplitude \( (J) \) as a function of the horizontal tune \( (Q) \). The 13/3 resonance line was used for the PEFP. When the amplitude is closer to zero, the tori become circular.

\[ \sqrt{2J} \sin(\phi) \]
\[ \sqrt{2J} \cos(\phi) \]

\[ \text{Fig. 1. (Color online) Betatron motion near the } 3\nu_x = 13 \text{ resonance at the RCS in PEFP (} x - P_x \text{ phase space).} \]

\[ \sqrt{2J} \sin(\phi) \]
\[ \sqrt{2J} \cos(\phi) \]

\[ \text{Fig. 2. (Color online) Betatron motion near the } 3\nu_x = 13 \text{ resonance at the RCS in PEFP (} \phi - J \text{ phase space).} \]

\[ \text{Fig. 3. The amplitude } (J) \text{ as a function of the horizontal tune } (Q). \]

\[ \text{Fig. 4. (Color online) Layout of the RCS in the PEFP. The slow extraction line exists inside the red round line. Four sextupoles to generate the third-integer resonance were located at the S2 positions. Also, six sextupoles for the chromaticity correction were located the positions C1, C2, and C3.} \]

**III. DESIGN OF THE SLOW BEAM EXTRACTION LINE FOR THE RCS**

The third-integer resonance mechanism due to the sextupoles at the RCS was used for slow extraction in the PEFP. The slow extraction line was located in a long straight section. The line consists of BMs (bump magnets), ESS (electrostatic septum) and SMs (septum magnets). The RCS of the PEFP has \( 2.5 \times 10^{13} \) protons per pulse and 60-kW full-beam power at 1 GeV. The RCS circumference is 222.16 m with a four-fold symmetry period. The details of the characteristic of the slow extraction line are as follows [3]:

- The length of the slow extraction line in the long straight section is 20.94 m.
Table 1. The Characteristics of the six sextupoles to correct the chromaticities.

<table>
<thead>
<tr>
<th>Location Name</th>
<th>Value</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>SXF3: Focusing sextupole</td>
<td>L = 0.2 m, K2 = 0.792082 m⁻³</td>
<td>C1</td>
</tr>
<tr>
<td>SXD1: Defocusing sextupole</td>
<td>L = 0.2 m, K2 = -2.38259 m⁻³</td>
<td>C2</td>
</tr>
<tr>
<td>SXF4: Focusing sextupole</td>
<td>L = 0.2 m, K2 = 0.792082 m⁻³</td>
<td>C3</td>
</tr>
</tbody>
</table>

Table 2. PEFP RCS tunes and chromaticities with and without an orbit deformation at the ESS.

<table>
<thead>
<tr>
<th>Bump Amp (mm)</th>
<th>Qₓ</th>
<th>Qᵧ</th>
<th>ξₓ</th>
<th>ξᵧ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.333</td>
<td>4.277</td>
<td>0.0019646</td>
<td>0.300778</td>
</tr>
<tr>
<td>44</td>
<td>4.333</td>
<td>4.277</td>
<td>-0.421865</td>
<td>-0.133906</td>
</tr>
</tbody>
</table>

Table 3. Twiss parameters at the ESS and the sextupole magnets.

<table>
<thead>
<tr>
<th>Location Name</th>
<th>βₓ</th>
<th>βᵧ</th>
<th>µₓ</th>
<th>µᵧ</th>
<th>αₓ</th>
<th>αᵧ</th>
<th>x</th>
<th>x'</th>
<th>Dₓ</th>
<th>Dᵧ</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESS</td>
<td>19.592</td>
<td>5.459</td>
<td>1.084</td>
<td>1.062</td>
<td>-2.029</td>
<td>0.604</td>
<td>42.227</td>
<td>4.551</td>
<td>-0.124</td>
<td>-0.020</td>
</tr>
<tr>
<td>SFX21</td>
<td>16.979</td>
<td>5.483</td>
<td>0.153</td>
<td>0.180</td>
<td>-1.654</td>
<td>0.595</td>
<td>6.521</td>
<td>0.598</td>
<td>-0.552</td>
<td>-0.040</td>
</tr>
<tr>
<td>SFX22</td>
<td>16.299</td>
<td>5.681</td>
<td>1.240</td>
<td>1.248</td>
<td>-1.600</td>
<td>0.680</td>
<td>0.978</td>
<td>0.488</td>
<td>-0.087</td>
<td>-0.008</td>
</tr>
<tr>
<td>SFX23</td>
<td>16.058</td>
<td>5.551</td>
<td>2.323</td>
<td>2.317</td>
<td>-1.622</td>
<td>0.671</td>
<td>3.999</td>
<td>0.713</td>
<td>-0.311</td>
<td>-0.031</td>
</tr>
<tr>
<td>SFX24</td>
<td>16.859</td>
<td>5.429</td>
<td>3.407</td>
<td>3.386</td>
<td>-1.702</td>
<td>0.629</td>
<td>6.066</td>
<td>0.757</td>
<td>-0.442</td>
<td>-0.042</td>
</tr>
</tbody>
</table>

Table 4. Input parameters of the bump magnets and the ESS for the MAD8.

<table>
<thead>
<tr>
<th>Magnets</th>
<th>Type</th>
<th>Length</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUMP1</td>
<td>KICKER</td>
<td>0.5 m</td>
<td>0.023 rad</td>
</tr>
<tr>
<td>BUMP2</td>
<td>KICKER</td>
<td>0.5 m</td>
<td>-0.023 rad</td>
</tr>
<tr>
<td>BUMP3</td>
<td>KICKER</td>
<td>0.5 m</td>
<td>-0.023 rad</td>
</tr>
<tr>
<td>BUMP4</td>
<td>KICKER</td>
<td>0.5 m</td>
<td>0.023 rad</td>
</tr>
<tr>
<td>ESS</td>
<td>ELSEPARATOR</td>
<td>0.5 m</td>
<td>0.715 MeV/m</td>
</tr>
</tbody>
</table>

- The bump magnets, ESS and septum magnets were installed in the slow extraction line.
- The long straight section has high a βₓ and a low αₓ.
- The η and the η' are zero in the long straight section.

The horizontal betatron tune, νₓ, approaches νₓ = 13/3 resonance line. Six sextupole magnets (Table 1) in the C1, C2, and C3 positions (Fig. 4) were used for horizontal and vertical chromaticity corrections. The remaining four sextupole magnets in the S2 positions were used to generate the third-integer resonance.

IV. TWISS PARAMETER OF THE RCS IN THE PEFP

Twiss parameters at the locations of the sextupoles and at the electrostatic septum were obtained by using MAD8 [4]. The parameters in the tables were obtained from simulations under different initial conditions. Table 2 gives the tunes and the chromaticities obtained with and without an orbit deformation at the electrostatic septum. The shift in the chromaticity due to an orbit deformation is very small.

Table 3 shows the Twiss parameters at the ESS and the sextupole magnets. The electrostatic septum was placed at a high βₓ and low αₓ position. The sextupole magnets to generate the third-integer resonance were placed in the last part of the long straight section for each period.

V. BEAM TRACKING FOR SLOW BEAM EXTRACTION AT RCS

Four bump magnets were installed in the long straight section. The closed orbit was shifted outward 44 mm in the x-direction due to four bump magnets. The ESS was installed 42 mm outward in the x-direction. Table 4 shows the input parameters of the bump magnets and the ESS for the MAD8 [4]. These parameters were used
for single particle tracking simulation at the RCS. The single particle tracking simulation depends on signs of the sextupole magnets.

Figure 6 shows the result of the simulation for a combinations of different signs of the sextupole magnets. The (+, −, −, +) sign order for slow beam extraction was selected because of the ESS installed at $x = 42$ mm. The initial conditions for the MAD8 tracking simulation at the ESS were chosen at $x = 1.45$ mm and $P_x = -0.65$ mrad, with a momentum spread of 0.3% and 1000 turns.

Figure 7 shows the single particle motion in $x - P_x$ phase space at the ESS entrance. The ESS length was assumed to be 0.5 m, and the strength was assumed to be 0.715 MeV/m. The result of the tracking simulation shows that particle moves along the separatrix branch. When the particles moves to 42 mm, the particle will be extracted to the external beam line by the ESS.

Figure 8 shows the orbit of a circulating beam at the RCS. The orbit was shifted to 44 mm due to the bump magnets, and the separated beam of the ESS is extracted along the external beam line by using the septum magnets. Two types of septum magnets were installed downstream of the ESS in the slow extraction line [5].

Figure 9 shows the scheme of the designed slow beam extraction line. The first type of septum magnet was designed with a 0.5-m length and a 0.2-rad angle in the MAD8. The second type of septum magnet was designed with a 1-m length and a 0.5-rad angle. A beam trace simulation due to septum magnets at the slow beam extraction line was performed. The beam separated by the ESS leaves the ring vacuum chamber due to the septum.
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VI. SUMMARY

The non-linear beam dynamics due to sextupole magnets at the RCS in the PEFP was investigated. The effects of the third-order resonance were examined, and a slow extraction system with sextupoles was designed. The slow extraction from the RCS makes use of the sextupole magnets that drive the third integer resonance controlled by tune ramping. A single particle tracking simulation was performed to observe the single particle motion in phase space. The beam trace simulation at the RCS shows that the beam paths of the extracted particles are stable. The simulations and calculations, that the slow extraction system had been optimized.

REFERENCES