Investigation of the effect of space charge in compact-Energy Recovery Linac

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- Summary
The generation of ultra-low emittance beams is need to demonstrate before constructing 3GeV ERL.

Compact-ERL

**2011**
- 4.2 GeV, 10 mA, 1 mm-mrad

**2012**
- 35 MeV, 10 mA, 1 mm-mrad

**3GeV ERL**
- 77 pC, 0.1 mm-mrad
- 100 mA, 77 pC, 1 mm-mrad
- 10 mA, 7.7 pC, 0.1 mm-mrad
Longitudinal Space Charge Effect

Initial beam distribution (Beer-can shape)

Electron

Electric field

Electric field

\[ E_z(z) = \frac{Q}{2\pi\varepsilon_0 a^2} \left[ -\frac{1}{2} \frac{z}{L} + \frac{1}{2} + \frac{z}{L} + \sqrt{\left(\frac{z}{L} - \frac{1}{2}\right)^2 + A^2} - \sqrt{\left(\frac{z}{L} + \frac{1}{2}\right)^2 + A^2} \right] \]

Electric field induced by Many electrons

The head and tail of bunch was accelerated and decelerated, respectively

Energy spread growth

where \( L = \sqrt{12\sigma_z} \): Full length of bunch
\( A = \alpha/\gamma L \), \( Q \) is charge
\( \alpha \) is radius, \( \varepsilon_0 \) is permittivity.
Longitudinal Space Charge Effect

\[ \Delta E = k_0 + k_1 s + k_2 s^2 + k_3 s^3 \]

Fitted parameters

- Initial energy spread: \( k_0 = 3.49 \pm 0.03 \)
- 1st order SC effect: \( k_1 = 0.0956 \pm 0.0335 \)
- 2nd order SC effect: \( k_2 = 0.0234 \pm 0.00943 \)
- 3rd order SC effect: \( k_3 = -0.000635 \pm 0.000721 \)

35 MeV, 8.54 m Drift space

GPT (3D SC)

Fitted curve

Analytic Model
Longitudinal Space Charge Effect

\[ \Delta E = k_0 + k_1 s + k_2 s^2 + k_3 s^3 \]

Fitted parameters:
- Initial energy spread: \( k_0 = 0.453 \pm 0.0945 \)
- 1st order SC effect: \( k_1 = 3.20 \pm 0.109 \)
- 2nd order SC effect: \( k_2 = 0.0924 \pm 0.033 \)
- 3rd order SC effect: \( k_3 = -0.020 \pm 0.0027 \)

Energy [MeV] vs. Bunch Length [m]

- 5 MeV, 8 m Drift space

\( \Delta E \) [keV] vs. Bunch length [mm]
SC effect in merger section

Uniform Energy spread ($\delta_0$) $x'$

LSCF Energy spread ($\delta(s)$)

see for example, B.E. Carlsten et al., IEEE QE 27, 2580 (1991).
SC effect in merger section

Analytical Calculation

First order theory

\[
x'' + \frac{x}{\rho^2} = \frac{1}{\rho}(\delta_0 + \delta_{SC} + k_1(s - s_0) + k_2(s - s_0)^2 + k_3(s - s_0)^3)
\]

\[
\begin{pmatrix}
x_f \\
x'_f \\
\delta_0 \\
\delta_{SC} \\
k_{1f} \\
k_{2f} \\
k_{3f}
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \rho \sin \theta & \rho(1 - \cos \theta) & \rho(1 - \cos \theta) & \rho^2(\theta - \sin \theta) & \rho^3(\theta^2 + 2 \cos \theta - 2) & \rho^4(\theta^4 + 6 \sin \theta - 6 \theta) \\
-\frac{1}{\rho} \sin \theta & \cos \theta & \sin \theta & \sin \theta & \rho(1 - \cos \theta) & 2 \rho^2(\theta - \sin \theta) & 3 \rho^3(\theta^2 + 2 \cos \theta - 2) \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho \theta & (\rho \theta)^2 & (\rho \theta)^3 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x_i \\
x'_i \\
\delta_i \\
\delta_{SC} \\
k_{1i} \\
k_{2i} \\
k_{3i}
\end{pmatrix}
\]
SC effect in merger section

Energy: 5 MeV   \( \varepsilon_{nx} = 0.1 \text{ mm-mrad} \)
Charge: 77 pC   \( \varepsilon_{ny} = 0.1 \text{ mm-mrad} \)
\( \Delta E/E_0 \): 0.01 \%
Bunch length (rms): 0.9 mm (3 ps)
Macro-particles: 10 k

Horizontal CS parameter scan

-0.35 rad
1.187 mm-mrad

Vertical CS parameter scan

-0.62 rad
0.784 mm-mrad

\( x', x \) Space charge kick

Angle of ellipse
SC effect in merger section

Analytical calculation

Numerical calculation

Normalized Emittance

\[ \varepsilon_{nx} = \left( (\varepsilon_0 \beta + D^2)(\varepsilon_0 \gamma + D'^2) - (\varepsilon_0 \alpha_x - DD')^2 \right) \gamma_E \]

where \((D, D') = \Delta k_{rms} (\zeta_x, \zeta_x'), \Delta k_{rms} = 0.207 \% /m\)

see for example, V.N. Litvinenko, R. Hajima and D. Kayran, NIM-A557, 165-175 (2006)
Effect of the energy spread

The merger has the following momentum compaction factors:

\[ R_{56} = -20.54 \text{mm} \quad T_{556} = 65.81 \text{mm} \]

A path length difference for particles with a relative energy deviation \( \delta \) is given by:

\[ \Delta z = \eta \delta = R_{56} \delta + T_{566} \delta^2 + U_{5666} \delta^3 \ldots \]

Therefore, the growth of emittance in the merger section can be reduced by the changing of the energy spread of injected beam.
Effect of the energy spread

\[ d_z = \frac{\sigma_E}{\sigma_z} = \frac{25 \text{keV}}{0.9 \text{mm}} \]

\[ = -27.78 \text{ [MeV/m]} \]

Energy : 5 MeV
Charge : 77 pC
\( \varepsilon_{nx} = 0.1 \text{ mm-mrad} \)
\( \varepsilon_{ny} = 0.1 \text{ mm-mrad} \)
Bunch length(rms) : 0.9 mm (3 ps)
Macro-particles : 10 k
Energy spread : 0.1 %

A path length difference due to the energy spread in merger

\[ \Delta z = R_{56}\delta + T_{566}\delta^2 \approx 0.1\text{mm} \]

\[ R_{56} = -20.54\text{mm} \]
\[ T_{556} = 65.81\text{mm} \]
Effect of the energy spread

Energy: 5 MeV  \( \varepsilon_{nx} = 0.1 \text{ mm-mrad} \)
Charge: 77 pC  \( \varepsilon_{ny} = 0.1 \text{ mm-mrad} \)
Bunch length (rms): 0.9 mm (3 ps)
Macro-particles: 10 k

\[
d_z = \frac{\sigma_E}{\sigma_z} = -40 \sim 40 \text{ [MeV/m]}
\]

Horizontal emittance \( \sim 0.735 \text{ mm-mrad} \)
Effect of the energy spread
Summary

The study of the space charge effect in the merger section of compact-ERL was performed to preserve the ultra-low transverse emittance in the ring.

In the very low energy case, around 5 MeV, the effect of changing of the length of bunch due to the SC effect was observed.

The minimum emittance, 0.78 mm-mrad, was achieved by changing of the Courant-Snyder parameter in merger section.

The effect of the energy spread of injected beam was investigated. It is one of the useful parameter for minimization of the growth of transverse emittance in merger section.

Thank you for your attention!