Emittance Minimization by Courant-Snyder Parameter Scan in Merger Section at the Compact Energy Recovery Linear Accelerator

Ji-Gwang Hwang, Eun-San Kim
Kyungpook National University, 1370 Sankyok-dong, Buk-ku, Daegu, 702-701, Korea
Tsukasa Miyajima
KEK, Tsukuba, Ibaraki 305-0801, Japan

Abstract
The project of compact-Energy Recovery Linac (c-ERL) at Photon Factory in KEK is a test facility for the 5 GeV ERL, which is one of the candidates of next generation light source. It consists of injector system, merger section, main SRF section, return arc, long straight section and beam dump. The injector system produces beams with a low-energy of 5 MeV and low-emittance less than 1 mm rad. It causes the large emittance growth by space charge force in merger section, which consists of two rectangular type dipole magnets and one sector type magnet. Dispersion also causes the displacement of bunch slice on horizontal plane. The displacement of bunch slice is laid on the kick angle induced by space charge force. Also, each slice has the orientation which consists of two rectangular type dipole magnets and one sector type magnet. Dispersion also causes the displacement of generation light source. It consists of injector system, merger section, main SRF section, return arc, long straight section and beam dump. The project of compact-energy recovery linear accelerator is given by

\[ \epsilon = \left( \epsilon_0 + D' \right) \left( \epsilon_1 + D'' \right) \]
where \( \epsilon_0 \) and \( \epsilon \) are the initial and final emittance as un-normalized values, respectively.

1. Merger section
Many users require a fourth generation light source that can produce high charge, short pulse, low transverse emittance and a high peak current electron beams. The Energy Recovery Linac is one of the candidates for the fourth generation light sources that can meet these requirements. The compact-ERL at KEK, in the final stage, will provide a beam energy of around 125 MeV and a bunch charge of 77 pC, which is a prototype for the future 5 GeV ERL at KEK. The c-ERL consists of an injector system, a merger section, a superconducting RF (SRF) section, two return loops and two straight sections. In the early commissioning phase, the injector produces electron beams with a bunch charge of 77 pC, beam energy of 5 MeV and bunch length of 0.6 mm rms. The beam energy is increased by 30 MeV with two cell SRF cavities. The energy spread induced in an achromatic cell results in the growth of projection emittance at the exit of the achromatic cell. A merger section with 3-dipole was adopted for the flexible beam transport of the high energy circulating beam. The layout of the 3-dipoles merger is shown in Fig. 1.

![Figure 1: Layout of a merger section.](image)

As shown in Fig. 1, the bending magnet at the center of the merger is sector type and bending magnets at the entrance and exit of the merger are rectangular type. The center bending magnet have an edge angle to achieve zero dispersion at the exit of the merger section. Secondly, the analytical calculation of the emittance growth in the merger section was performed by using the first-order theory.

2. First order theory
The emittance growth due to the displacement of the bunch slices in phase space can be minimized by matching the displacement to the orientation of the phase ellipse at the exit of merger. We present the results of the emittance minimization performed by matching of the angle of the phase ellipse by scan of CS (Courant-Snyder) parameter.

![Figure 2: Growth of projected emittance due to the SC effect in merger section.](image)

(a) Maximum case of projected emittance growth
(b) Minimum case of projected emittance growth

Therefore, the angle of the displacements due to the SC effect is given by \( \phi = \tan(\epsilon_0/\gamma) \). The analytical calculation of the space charge dispersion requires the space charge kick angle. The result of the space charge dispersion in the merger section is shown in Fig. 3.

![Figure 3: Space charge dispersion at the merger section.](image)

3. Analytical calculation
In the analytical calculation, by using the first-order theory, the transfer matrix for each element is derived by Green’s function method. Based on the first-order theory, the transverse emittance growth in the merger section is given by \( \epsilon = (\epsilon_0 + D)(\epsilon_1 + D') \) where \( \epsilon_0 \) and \( \epsilon \) are the initial and final emittance as un-normalized values, respectively. The vertical CS parameter was changed to investigate the effect of coupled motion. In the calculation, the horizontal CS parameter was fixed to \( \beta_0 = 5.5 \text{ m} \) with \( \alpha_0 = -1.6 \) and shows a minimum emittance growth.

![Figure 4: Analytical calculation results of emittance as function of twist parameter at the entrance of merger](image)

![Figure 5: Transverse emittance at the exit of the merger as function of the initial xi and xi.](image)

4. Numerical calculation
The numerical calculation is done by using the code of General Particle Tracer (GPT), which includes the calculation of 3-dimensional SC force with actual electric and magnetic fields. In the particle tracking simulation by using GPT code, the initial normalized transverse emittance is 0.1 mm rad, the bunch length is 3 ps (rms), the beam energy is 5 MeV and the the bunch included the particle distribution of 10000 macroparticles. The bunch distribution was assumed to be beren-para shape.

![Figure 6: Transverse emittance at the exit of merger section as a function the orientation of the phase ellipse.](image)

We found a minimum transverse emittance growth of 1.09 mm rad in the merger section when the vertical CS parameter was fixed to \( \beta_0 = 9 \text{ m} \) with \( \alpha_0 = 0 \). Also, the effect of coupled motion of the beam was investigated to minimize the emittance growth in the merger section. The vertical CS parameter was changed to investigate the effect of coupled motion. In the calculation, the horizontal CS parameter was fixed to \( \beta_0 = 5.5 \text{ m} \) with \( \alpha_0 = -1.6 \) and shows a minimum emittance growth.

![Figure 7: The change of transverse emittance at the exit of merger section as function the orientation of the phase ellipse due to the change of vertical CS parameters.](image)

5. SUMMARY
Based on the first-order theory, the emittance growth due to the displacement of bunch slices in phase space was minimized by matching the orientation of the phase ellipse to the kick angle induced by the SC force. The minimum horizontal emittance growth at the exit of the merger becomes 0.733 mm rad.